A critical review of robust self-scheduling for generation companies under electricity price uncertainty

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\textbf{ABSTRACT}

For a generation company trading in an electricity market, efficient control of the financial risks and robustness is as vital as maximizing profit. A robust approach is preferred since the generation company can obtain an optimal self-schedule considering price volatility as a source of uncertainty. The goal of this paper is to implement and compare different robust approaches such as robust optimization methods with different uncertainty sets, conditional value-at-risk based stochastic programming, and information gap decision theory for self-scheduling of generation companies. Moreover, all robust methods are applied to test cases with different price behaviors in the long-run to demonstrate the performance and features of each method. Finally, the different self-scheduling strategies based on the price data and the generation company’s desired robustness level are proposed.

\section{Introduction}

\subsection{Background and motivation}

In a competitive market environment, a generation company (GenCo), as a decision-maker, not only tries to attain a profitable bidding strategy, but also strives to achieve a robust position (i.e. a strategy hedged against any realization of the uncertainty as the difference between the forecasted and actual values). The self-scheduling of a GenCo is a complex and difficult optimization problem, not only due to the need for meeting all equality and inequality constraints of the generating units during the entire scheduling period, such as minimum on/off duration, generation capacity limits, ramping up/down limits of generating units, but also due to all issues affecting electricity market prices and increasing their volatility, such as system load forecasting, predicting rival GenCos bidding strategies, and transmission congestion. In other words, electricity market prices and their volatility are the key factors complicating the self-scheduling problem of a GenCo. The price signal forces the on/off status of a generation unit and its volatility significantly affects the self-scheduling problem of a GenCo. Since the forecasted electricity market prices are subject to uncertainty due to their high volatility\textsuperscript{[1]}, it is necessary to characterize the uncertainty of the forecasted electricity market prices aiming at hedging the self-schedules of GenCos against different realizations of uncertain electricity market prices. In other words, a GenCo should adopt optimization methods considering uncertainty for its self-scheduling approach. This has led to a growth in non-deterministic self-scheduling methods. These methods include robust optimization (RO) with different uncertainty sets\textsuperscript{[2–5]}, conditional value-at-risk based stochastic programming (CVaR-SP)\textsuperscript{[6,7]}, and information gap decision theory (IGDT)\textsuperscript{[8,9]}.

The RO methodology models uncertainty sets as bounded intervals, such as box, ellipsoidal, and polyhedral uncertainty sets\textsuperscript{[10]}. The ellipsoidal uncertainty set has been applied to the self-scheduling problem leading to a second-order cone model in\textsuperscript{[2]}. An ellipsoidal RO method has also been used in\textsuperscript{[3]} to determine the worst-case robust profit of the self-scheduling problem. Besides, there is some research work concentrated on the combination of the uncertainty sets with each other. The combination of the box and polyhedral uncertainty sets for RO as a linear optimization framework has been presented in\textsuperscript{[11]}. Moreover, this combined uncertainty set has been used to construct the offer curve of a generation company in\textsuperscript{[4]}. The RO approach presented in\textsuperscript{[11]} has been implemented to construct the bidding strategy of a wind farm and energy storage devices in\textsuperscript{[5]}. In addition, the combined box and ellipsoidal uncertainty set for RO has been introduced in\textsuperscript{[12]}. In RO methods including various uncertainty sets, a decision-maker can change the robustness of the solution by changing a specific parameter named the degree of robustness (DR).
### Nomenclature

**Function**

\[ f_u(\cdot) \]

generation cost function of unit \( u \)

**Parameters**

\[ \text{CSC}_u \]

cold startup cost of unit \( u \) ($)

\[ \alpha \]

coefficient of the piecewise linear generation cost function of unit \( u \)

\[ a_u, b_u, c_u \]

coefficients of the quadratic generation cost function of unit \( u \)

\[ B_u \]

binary parameter for unit \( u \) in hour \( t \), which is zero if \( e_u = e_{u'} \), and one otherwise

\[ \text{Diff}_{\text{max}}, \text{Diff}_{\text{min}} \]

difference between maximum and minimum prices of each price interval in hour \( t \) ($/MWh)

\[ \text{Diff}_{\text{max}}^{\text{diff}}, \text{Diff}_{\text{min}}^{\text{diff}} \]

difference between maximum and minimum deterministic profits of each profit interval ($)

\[ E_u \]

predicted electricity price for hour \( t \) ($/MWh)

\[ E_{\text{up}}, E_{\text{down}} \]

electricity price of hour \( t \) in scenario \( s \) ($/MWh)

\[ E_{\text{up}}^{\text{max}}, E_{\text{up}}^{\text{min}} \]

upper and lower limit of \( E_u \) ($/MWh)

\[ \text{HSC}_u \]

hot startup cost of unit \( u \) ($)

\[ i, j \]

loop counters

\[ J \]

set of uncertain electricity market prices

\[ |U| \]

number of elements for the uncertainty set \( J \)

\[ \text{MD}_{\text{up}}, \text{MU}_{\text{up}} \]

minimum down and up time of unit \( u \), respectively (h)

\[ N_u \]

cold startup cost of unit \( u \) ($)

\[ N_i \]

number of price intervals

\[ N_{ij} \]

number of blocks of the piecewise linear generation cost function of unit \( u \)

\[ N_s \]

number of scenarios

\[ p_{\text{up}}^{\text{max}}, p_{\text{up}}^{\text{min}} \]

upper and lower limit of block \( i \) of the piecewise linear generation cost function of unit \( u \) (MW)

\[ p_{\text{down}}^{\text{max}}, p_{\text{down}}^{\text{min}} \]

upper and lower limit for unit \( u \), respectively (MW)

\[ S_u \]

slope of block \( i \) of the piecewise linear generation cost function of unit \( u \)

\[ RD_{\text{up}}, RU_{\text{up}} \]

ramp down and ramp up limit of unit \( u \), respectively

\( (\text{MW}/\text{h}) \)

\( SDR_u, SUR_u \)

shutdown and startup ramp limit of unit \( u \), respectively

\( (\text{MW}/\text{h}) \)

\( SU_u, T_{\text{up}} \)

startup cost of unit \( u \) after \( T \) hours down time ($)

\( \text{Hor}_{\text{up}} \)

required time to cool down unit \( u \) (h)

\( T \)

total hours of the scheduling period

\( U \)

total number of units

\( \Psi_u \)

degree of robustness for the box uncertainty set

\( \Psi_{\text{Ell}} \)

degree of robustness for the ellipsoidal uncertainty set

\( \Psi_{\text{Poly}} \)

degree of robustness for the polyhedral uncertainty set

\( \alpha \)

an a non-negative weight factor that weighs conditional robust profit against expected profit

\( \lambda \)

degree of uncertainty

\( \alpha \)

per unit confidence level

\( \sigma \)

profit deviation factor

### Variables

\[ h_u, q_u, v \]

continuous auxiliary robust modeling variables

\[ m_{\text{uu}} \]
	non-negative auxiliary variable used for modeling non-decreasing constraints

\[ p_u \]

power offered by unit \( u \) in hour \( t \) for energy auction; \( p_{\text{act}} \) is \( p_u \) value in scenario \( s \) (MW)

\[ p_{\text{bid}} \]

power in block \( i \) of the piecewise linear generation cost function of unit \( u \) in hour \( t \) (MW)

\[ r_u^{\text{u}}, r_u^{\text{d}} \]

free auxiliary variable used for modeling non-anticipativity constraints

\[ w_u \]

startup cost of unit \( u \) in hour \( t \) ($)

\[ x_u, y_u \]

binary variables indicating startup and shutdown status of unit \( u \) in hour \( t \), respectively

\[ z_u \]

binary variable indicating status of unit \( u \) in energy auction in hour \( t \) (1/0 for accepted/not-accepted)

\[ \phi_u \]

cold uncertainty parameter for information gap decision theory

\( \theta \)

uncertainty parameter for information gap decision theory

### 1.2. Contributions

The main contributions of this paper are:

1. The mathematical formulations of different robust approaches including Box RO (BRO), Ellipsoidal RO (ERO), Polyhedral RO (PRO), Box and Ellipsoidal RO (BERO), Box and Polyhedral RO (BPRO), CVaR-SP, and IGDT models are proposed. Also, the characteristics of the uncertainty sets corresponding to BRO, ERO, PRO, BERO, and BPRO are presented by means of relevant theorems and proofs.

2. Various self-scheduling strategies based on the robust approaches are proposed for GenCos to participate in an electricity market considering the price data and desired robustness level.

3. To correctly analyze and compare the performance of these robust methodologies in the uncertain environment of self-scheduling, a
post-optimization procedure is proposed. This procedure evaluates the long-run performance of the robust methodologies encountering different realizations of uncertain electricity prices.

1.3. Assumptions

For simplicity and better illustration of the underlying ideas of the robust approaches in the self-scheduling models presented in this paper, the following assumptions are made:

1. The GenCo is a price taker, in which the GenCo must accept prevailing prices (i.e. market clearing prices) in a market without the capability of changing it.
2. Only energy auction is taken into account.
3. Shutdown costs of units are neglected.

These assumptions are in line with many other self-scheduling research work such as [1–7,17,18].

1.4. Paper organization

The rest of the paper is organized as follows. In Section 2, the deterministic self-scheduling model is first described briefly. Then, non-deterministic self-scheduling models based on the robust methodologies are introduced in Section 3, a comprehensive comparison amongst all robust self-scheduling models using real-world electricity market prices is presented. Finally, concluding remarks are provided in Section 4.

2. Deterministic and non-deterministic self-scheduling models

2.1. Deterministic self-scheduling model

The deterministic self-scheduling model based on the mentioned assumptions can be formulated as a mixed-integer linear programming (MILP) problem given in (1)–(9) [19]:

\[
\max_\Omega \left[ \sum_{t=1}^{T} \sum_{u=1}^{U} (E_c - p_u) - \sum_{t=1}^{T} \sum_{u=1}^{U} u_u - \sum_{t=1}^{T} \sum_{u=1}^{U} f_u (p_u) \right]
\]

s.t.

\[
u_u \geq 0 \quad \forall u, \forall \tau = 1, \ldots, T
\]

\[
S_u = \begin{cases} 
CSC_u & \text{if } \tau > T_u^{old} + MD_u \\
HSC_u & \text{if } \tau \leq T_u^{old} + MD_u
\end{cases} \quad \forall u
\]

\[
\sum_{n=1}^{t} z_{an} \leq z_{at} \quad \forall u, \forall t
\]

\[
z_{a(t-1)} - x_u + x_{a(t-1)} = 0 \quad \forall u, \forall t
\]

\[
P_{u}^{min} - z_{a} \leq p_{u} \leq P_{u}^{max} + z_{a} \quad \forall u, \forall t
\]

\[
P_{d} \leq (P_{u} - X_{du}) \cdot R_{ud} \cdot x_{at} + SU_R \cdot x_{at} \quad \forall u, \forall t
\]

\[
P_{d} \leq P_{u} - (X_{du} - x_{a(t-1)}) \cdot R_{ud} \cdot z_{a} + SDR_{u} \cdot x_{at} \quad \forall u, \forall t
\]

In (1), \( \Omega \) includes the decision variables of the self-scheduling model: \( \Omega = [p_u, z_{at}, x_{at}, Y_{aj}] \), \( \forall u, \forall t \). The first, second and third terms of (1), denoted by \( DT \), indicate revenue from selling energy, startup cost and operation cost of GenCo, respectively. The objective function of (1) represents profit of GenCo in the energy auction. The generation cost functions of \( f_u(.) \) are linearized by the piecewise linear approximation of [20], which is given in Appendix A. Constraints (2) and (3) model startup costs of units including the costs of hot and cold starts. Minimum up and down time limits of units are modeled in (4)–(6) based on three-binary variable formulation (including \( x_u, y_{aj} \), and \( z_{at} \)), which is more effective than conventional one-binary variable formulation [17]. Unit generation limits are represented in (7). Ramp up and down rate limits of units considering startup and shutdown ramps are given in (8) and (9).

2.2. Non-deterministic self-scheduling models

To model electricity price uncertainty in the self-scheduling framework, various non-deterministic approaches comprising BRO, ERO, PRO, BERO, BPRO, CVaR-SP, and IGDT are proposed. These approaches provide robust solutions against the uncertainty source with the capability of modeling the solution robustness. Details of each non-deterministic self-scheduling model are presented in the following.

2.2.1. BRO-based self-scheduling model

In this approach, the uncertainty source is modeled using infinite-norm as a box uncertainty set (BS):

\[
BS = \{ \tilde{E} \in [E - \tilde{E}, E + \tilde{E}] \} \quad \forall j \in J
\]

where \( \| \frac{\tilde{E}}{E} \|_{\infty} = \max \left\{ \frac{E_1 - \tilde{E}_1}{E_1}, \frac{E_2 - \tilde{E}_2}{E_2}, \ldots, \frac{E_J - \tilde{E}_J}{E_J} \right\} \)

In our self-scheduling model, \( j = t \), thus:

\[
BS = \{ \tilde{E} \in [E - \tilde{E}, E + \tilde{E}] \} \quad \forall t
\]

The same value of box DR, i.e. the same value of \( \Psi_t \in [0,1] \), is adopted for all uncertain electricity prices to control the size of BS. DR in RO approaches gives the decision-maker the capability of changing the robustness of the RO solution.

To construct the BRO-based robust counterpart of the deterministic self-scheduling model (1)–(9), only (1) is changed since only (1) includes the uncertain variables:

\[
\max_{\Omega} \left[ \sum_{t=1}^{T} \sum_{u=1}^{U} (E_c - p_u) - \sum_{t=1}^{T} \sum_{u=1}^{U} u_u - \sum_{t=1}^{T} \sum_{u=1}^{U} f_u (p_u) \right]
\]

s.t.

\[
u_u \geq 0 \quad \forall u, \forall t \in 1, \ldots, T
\]

\[
S_u = \begin{cases} 
CSC_u & \text{if } \tau > T_u^{old} + MD_u \\
HSC_u & \text{if } \tau \leq T_u^{old} + MD_u
\end{cases} \quad \forall u
\]

\[
z_{a(t-1)} - x_u + x_{a(t-1)} = 0 \quad \forall u, \forall t
\]

\[
P_{u}^{min} - z_{a} \leq p_{u} \leq P_{u}^{max} + z_{a} \quad \forall u, \forall t
\]

\[
P_{d} \leq (P_{u} - X_{du}) \cdot R_{ud} \cdot x_{at} + SU_R \cdot x_{at} \quad \forall u, \forall t
\]

\[
P_{d} \leq P_{u} - (X_{du} - x_{a(t-1)}) \cdot R_{ud} \cdot z_{a} + SDR_{u} \cdot x_{at} \quad \forall u, \forall t
\]

Considering BS given in (11), the bi-level optimization problem of (12) can be reformulated as the single-level optimization problem of (13) [10]:

\[
\max DT - \Psi_t \sum_{t=1}^{T} \sum_{u=1}^{U} (E_c - p_u) \quad s.t. (2)-(9)
\]

where \( DT \) is as given in (1). The BRO-based self-scheduling model of (13) has MILP form.

2.2.2. ERO-based self-scheduling model

Through ERO, the ellipsoidal uncertainty set of electricity market prices, i.e. ES, is modeled using Euclidean norm (2-norm) as given below:

\[
ES = \{ \tilde{E} \in [E - \tilde{E}, E + \tilde{E}] \} \quad \forall j \in J
\]

where \( \| \frac{\tilde{E}}{E} \|_2 = \sqrt{\sum_{j=1}^{J} \left( \frac{\tilde{E}_j}{E_j} \right)^2} \).

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Hence,
\[
ES = \left\{ \tilde{E}_t \in [E_t - \tilde{E}_b, E_t + \tilde{E}_b] \mid \forall t: \sqrt{T} \left( \sum_{i=1}^{T} \left( \frac{\tilde{E}_t - E_t}{E_t} \right)^2 \right) \leq \Psi_E \right\}
\] (15)

In the ERO approach, the ellipsoidal DR of \( \Psi_E \in [0, \sqrt{T}] \) (i.e., \([0, \sqrt{T}]\)) controls the solution robustness. The robust counterpart of the ERO approach can be obtained from (12) by changing the argument of the ‘min’ operator from \( \tilde{E}_t \in BS \) to \( \tilde{E}_t \in ES \). Subsequently, the ERO-based robust counterpart can be constructed as follows [10]:
\[
\max \left[ DT - (\Psi_E) \sqrt{T} \left( \sum_{i=1}^{T} \left( \frac{\tilde{E}_t - E_t}{E_t} \right)^2 \right) \right] \quad \text{s.t.} \quad (2)-(9)
\] (16)

where \( DT \) is as given in (1). The ERO-based non-deterministic self-scheduling model of (16), which is a mixed-integer second-order cone programming (MISOCP) model with convex (hull) [21], is solved as a mixed-integer-constrained quadratic programming (MIQCP) problem [22].

2.2.3. PRO-based self-scheduling model

PRO uses 1-norm to formulate the polyhedral uncertainty set \( PS \) of electricity market prices as given in (17):
\[
PS = \left\{ \tilde{E}_t \in [E_t - \tilde{E}_b, E_t + \tilde{E}_b] \mid \forall j \in J: \left\| \frac{\tilde{E}_t - E_t}{E_t} \right\|_1 \leq \Psi_{PS} \right\}
\] (17)

where
\[
\left\| \frac{\tilde{E}_t - E_t}{E_t} \right\|_1 = \left\| \frac{\tilde{E}_t - E_t}{E_t} \right\| + \left\| \frac{\tilde{E}_t - E_t}{E_t} \right\| + \ldots + \left\| \frac{\tilde{E}_t - E_t}{E_t} \right\|_1 \quad \text{therefore}
\]

\[
PS = \left\{ \tilde{E}_t \in [E_t - \tilde{E}_b, E_t + \tilde{E}_b] \mid \forall t: \sqrt{T} \left( \sum_{i=1}^{T} \left( \frac{\tilde{E}_t - E_t}{E_t} \right) \right) \leq \Psi_{PS} \right\}
\] (18)

The polyhedral DR of \( \Psi_{PS} \in [0, \bar{p}] \) (i.e., \([0,T]\)) is used to control the solution robustness. Considering PS instead of BS in (12), the following PRO-based robust counterpart can be obtained [10]:
\[
\max \left[ DT - (\Psi_{PS}) \right] \quad \text{s.t.} \quad (2)-(9)
\] (19)

where \( DT \) is as given in (1). The PRO-based self-scheduling model of (19) and (20) has MILP form.

The last term within the brackets of (13), (16) and (19) represents the protection functions (PFs). By changing box DR, ellipsoidal DR, and polyhedral DR (i.e., \( \Psi_{BS}, \Psi_{ERO}, \) and \( \Psi_{PS} \)), the magnitude of the PF and so the solution robustness of the BRO-based, ERO-based, and PRO-based non-deterministic self-scheduling models, respectively, can be adjusted.

2.2.4. BERO-based self-scheduling model

BERO employs the intersection of box and ellipsoidal uncertainty sets, denoted by BES, presented in (21):
\[
BES = BS \cap ES = \left\{ \tilde{E}_t \in [E_t - \tilde{E}_b, E_t + \tilde{E}_b] \mid \forall t: \frac{\tilde{E}_t - E_t}{E_t} \leq \sqrt{T}, \psi_E \leq E_t \right\}
\] (21)

As shown in [23], the following property for infinite-norm and 2-norm of the n-dimensional vector \( X \) holds:
\[
|X|_\infty \leq |X|_2 \leq \sqrt{n} |X|_\infty
\] (22)

Similarly, for the uncertain electricity market prices during the scheduling period (i.e., \( \tilde{E}_t \) for \( t = 1, \ldots, T \)), it can be concluded that:
\[
\left\| \frac{\tilde{E}_t - E_t}{E_t} \right\|_\infty \leq \left\| \frac{\tilde{E}_t - E_t}{E_t} \right\|_2 \leq \sqrt{T} \left\| \frac{\tilde{E}_t - E_t}{E_t} \right\|_\infty
\] (23)

Also, \( \left\| \frac{\tilde{E}_t - E_t}{E_t} \right\|_\infty \leq \psi_E \) and \( \left\| \frac{\tilde{E}_t - E_t}{E_t} \right\|_2 \leq \psi_E \), as shown in (21). Accordingly, in the proposed BERO model, \( \psi_E \in [\psi_E, \psi_E, \sqrt{T}] \) where \( \psi_E \in [0,1] \).

The robust counterpart of BERO method, presented in (24) and (25), is obtained by replacing BS with BS \( \cap ES \) in (12). The last two terms of (24) are the combination of the last terms of (13) and (16); (25) presents the relationship among power and auxiliary variables of the PFs of BERO. The BERO model of (24) and (25) is an MISOCP problem with convex (hull) [21], which is solved as an MIQCP [22].
\[
\max \left[ DT - (\Psi_{Psi}) \right] \quad \text{s.t.} \quad (2)-(9)
\] (24)

\[
(\Psi_{Psi})_{\sum_{i=1}^{T} (q_i)} \leq (1 + \lambda) \sum_{i=1}^{T} \left( \sum_{j=1}^{T} (E_j, P_{du, j}) - \sum_{j=1}^{T} \sum_{i=1}^{T} f_{j, i} (P_{du, j}) \right)
\]
(29)

(2)-(6)
\[
(p_{\text{out}}-p_{\text{in}}) = r_{\text{out}}^w \sum_{i=1}^{N} B_{\text{out}}^w \quad \forall u, \forall t, \forall s, s' (34)
\]

\[
\beta_i \geq \sum_{i=1}^{N} \pi_i \left( \sum_{j=1}^{U} (E^j_w p_{\text{out}}) - \sum_{j=1}^{U} \sum_{i=1}^{T} f_i (p_{\text{in}}) \right) - \left[ \sum_{j=1}^{U} (E^j_w p_{\text{out}}) - \sum_{j=1}^{U} \sum_{i=1}^{T} f_i (p_{\text{in}}) \right] - \mu \beta_i \geq 0 \quad \forall s (35)
\]

The compact form of (29) is max \((1 + \lambda) E - \lambda CVaR\) obtained from max \([E \pm \lambda CVaR]\) where \(E\) is the expected profit, \(CRP\) is the conditional robust profit, and \(\lambda\) is a weight factor to model the financial risk of the self-scheduling problem. Higher/lower values of \(\lambda\) lead to more risk-averse/risk-seeking models. In this stochastic approach, the constraints (2)–(6) of the deterministic model are directly applied, while the scenario-based version of the constraints (7)–(9), in the form of (30)–(32), is considered. Constraints (33) and (34) present non-decreasing and non-anticipativity constraints, respectively, which are essential to obtain reasonable solutions in the SP model. In non-decreasing constraint (33), the non-negative auxiliary variable \(m_{\text{out}}^w\) is used to ensure that \(p_{\text{out}}\) is more than \(p_{\text{in}}\) if \(E_{\text{in}}\) is more than \(E_{\text{out}}\). In non-anticipativity constraint (34), the binary parameter \(\lambda_{\text{out}}^w\) is 0, if the realization of the uncertain price variables in hour \(t\) are the same in two scenarios \(s\) and \(s'\), otherwise 1. The free auxiliary variable \(r_{\text{out}}^w\) is used to ensure that \(p_{\text{out}}\) and \(p_{\text{in}}\) can take any value if \(r_{\text{out}}^w = 0\), otherwise \(p_{\text{out}} = p_{\text{in}}\). The CVaR constraint of the model is shown in (35). For the mathematical details of this approach, the interested reader can refer to [7]. The CVaR-SP model of (29)–(35) has MILP form.

### 2.2.7. IGDT-based self-scheduling model

The IGDT focuses on decision making under severe uncertainty. Under severe lack of information, uncertainty is considered as the gap between what is known (historical data) and what needs to be known (actual data) to make competent decisions [15]. There are different IGDT models such as Energy-bound, Minkowski-norm, Slope-bound, Fourier-bound, Hybrid info-gap, and so forth, which are quadratic or non-linear [15]. Since the envelope-bound model is a proficient linear and convex approach to characterize the forecast uncertainties [15], in line with previous research work in the power system area [8,9,24–27], this uncertainty model is used in this paper instead of other non-linear and non-convex ones to characterize the forecast uncertainties of electricity prices in the self-scheduling problem of GenCos. The uncertainty set of envelope-bound-based IGDT, denoted by \(IS\), is as follows:

\[
IS = \left\{ \tilde{E}_i : \frac{\tilde{E}_{i} - E_i}{E_i} \leq \varepsilon \quad \forall t \right\} (36)
\]

The IGDT includes robust and opportunistic approaches to quantify pernicious and propitious variations of uncertain variables by means of robustness and opportuneness immunity functions [15]. The robust approach in the self-scheduling problem of GenCos focuses on pernicious variations of uncertain electricity prices on the profit (i.e., the realized electricity prices are lower than their forecasted values) and finds the greatest variations of the uncertain electricity prices when the minimum profit is not lower than a critical value (with guarantee). The opportunistic approach in the self-scheduling problem of GenCos focuses on propitious variations of uncertain electricity prices on the profit (i.e., the realized electricity prices are higher than their forecasted values) and finds the lowest variations of the uncertain electricity prices when the maximum profit is higher than a target value (but without guarantee). This research focuses on the robust approach of IGDT, which a risk-averse decision-maker desires to use. The generic form of robust approach is as follows:

\[
\max \theta (37)
\]

S.t. minimum requirements are always satisfied

Thus, the robust IGDT model of the self-scheduling problem becomes as follows:

\[
\text{s.t.} \sum_{t=1}^{T} \sum_{u=1}^{U} ((1-\beta)E_{u} - \sum_{t=1}^{T} \sum_{u=1}^{U} u_{w}^{u}) - \sum_{t=1}^{T} \sum_{u=1}^{U} f_{i}(p_{u}) \geq (1-\sigma). DP, (2)-(9) (38)
\]

where DP is the deterministic profit obtained from (1)–(9). The IGDT model has a mixed-integer non-linear programming (MINLP) form.

### 2.3. Construction of hourly offer curve

The construction of hourly offer curve is designed based on the price forecast confidence intervals, since confidence intervals for price forecasts can capture price volatility better than point price forecasts [28]. Then, the GenCos can efficaciously control the profit volatility. For the sake of fair comparisons, the same historical prices are used to find \(E_{\text{inv}}^{\text{max}}\) and \(E_{\text{inv}}^{\text{min}}\) of the confidence intervals for all non-deterministic self-scheduling models presented in the previous section.

#### 2.3.1. RO-based hourly offer curve for models 1–5

1. Set initial values: \(i = 0, E_i = E_{\text{inv}}^{\text{max}} \forall t, DR = DR_{\text{max}}\). The highest DR is considered, since price forecast error typically occurs in all hours of a day [4,29,30].
2. Set \(\text{Diff}_{\text{inv}} = (E_{\text{inv}}^{\text{max}}-E_{\text{inv}}^{\text{min}})/N_t \forall t\).
3. Based on the models 1–5, solve the robust optimization approaches for the interval: \(E_i \in [E_{\text{inv}}^{\text{max}}-\text{Diff}_{\text{inv}}, E_{\text{inv}}^{\text{max}}] \forall t \{4,18\}.
4. \(i = i + 1\), if \(i < N_i\), go to Step 3, otherwise the offer curve is obtained.

#### 2.3.2. SP-based hourly offer curve for model 6

1. Set \(\text{Diff}_{\text{inv}} = (E_{\text{inv}}^{\text{max}}-E_{\text{inv}}^{\text{min}})/N_t \forall t \) and \(\Pi_t = 1/N_t \{31,32\}.
2. Set \(E_o = E_{\text{inv}}^{\text{max}}-\text{Diff}_{\text{inv}} \forall t \in N_t.
3. Solve the CVaR-SP model of (29)–(35) by means of \(E_{io} \forall t \forall s \in N_s\).

#### 2.3.3. IGDT-based hourly offer curve for model 7

1. Set \(j = 0, E_i = E_{\text{inv}}^{\text{max}} \forall t, \) also obtain \(DP_{\text{max}}\) and \(DP_{\text{min}}\) as deterministic profit for \(E_{\text{inv}}^{\text{max}}\) and \(E_{\text{inv}}^{\text{min}}\), respectively.
2. Set \(\text{Diff}_{\text{inv}} = (DP_{\text{max}}-DP_{\text{min}})/N_t\).
3. \(DP = DP_{\text{max}}\) and \(\sigma = ((j \times \text{Diff}_{\text{inv}})/DP_{\text{max}}\).
4. Solve the IGDT model of (37) and (38) for \(\sigma\).
5. \(j = j + 1, \) if \(j < N_j, \) go to Step 3, otherwise the offer curve is obtained.

#### 2.4. Post-optimization procedure

To correctly and practically assess the efficiency of the non-deterministic self-scheduling approaches, their performance should be evaluated in the real market environment, i.e. with the realized market prices. To address this issue, a post-optimization procedure (POP) is proposed in this paper to determine the actual profit of each approach in different real-market-environment cases. After constructing the hourly offer curve of every method, the real clearing prices are used in the POP to determine the actual profit associated with each offer curve. For every hourly offer curve, the real clearing price of the market determines the accepted power from the GenCo, and thereby its profit for that hour.

For clarity, the schematic of the proposed self-scheduling
framework including pre-optimization, optimization, and post-optimization procedures, is illustrated in Fig. 1.

3. Case studies

In this section, all non-deterministic self-scheduling methods discussed in the previous section are implemented on the IEEE 30-bus [33] and IEEE 118-bus test systems [34] (the generator data for two test systems is presented in Appendix B). It is assumed the GenCo has 6, and 54 thermal generating units in the first and second test systems, respectively, and that the data obtained from [33,34] pertains to these units. Real price data for four months, including January, April, July, and October of the years 2010–2013 in the electricity market of New York Independent System Operator (NYISO) [35] is used as the historical data to determine the interval \([E_{\text{min}}, E_{\text{max}}]\). Additionally, the real price data of these four months of the year 2014 in NYISO electricity market is used as the clearing price for the offer curves. Thus, the test period, i.e. the four months of the year 2014, is different from the setup period. Accordingly, the performance of each non-deterministic self-scheduling method is evaluated by the out-of-sample data. For each hour of the test period, i.e. the four months of the year 2014, the obtained interval \([E_{\text{min}}, E_{\text{max}}]\) is shifted such that its mean value becomes the price forecast of that hour [1]. Every non-deterministic self-scheduling model uses the shifted price interval to construct its hourly offer curve as described in the previous section. Moreover, \(N_i = 100, N_i = 100, N_j = 100\) are considered for RO approaches, CVaR-SP model and IGDT method, respectively. In line with previous research work in the area, such as [4], \(N_i\) is considered equal to 100. And for the sake of fair comparisons, \(N_i\) and \(N_j\) are also considered equal to 100. In this case, the offer curves of all robust approaches would have 100 blocks. In CVaR-SP model, \(\alpha = 0.99\) and \(\lambda = 100\) are adopted [67]. All numerical experiments of this paper have been performed by CPLEX solver (for MILP and MISOCP models) and BARON solver (for MINLP models) in GAMS environment [36] on a 64-bit Windows-based server with 60 GB of RAM and 24 Intel Xeon processors clocking at 3.33 GHz.

It is assumed in the RO models, \(\text{DR} = \text{DR}^{\text{max}}\) in the numerical experiments of this section based on the reason described in the previous section.

Tables 1 and 2 present the POP results of the deterministic and robust approaches for the four months of the year 2014 in NYISO. In January, July, and October there are 744 (24 \(\times\) 31), and in April there are 720 (24 \(\times\) 30) hourly offer curves. It is seen that for \(\text{DR} = \text{DR}^{\text{max}}\) the profits of BRO, BERO, and BPRO become the same (the proof is provided in Appendix C). Moreover, for the same DR, ERO is less conservative than PRO and BRO is less conservative than ERO (the proof is provided in Appendix D). In each column of Tables 1 and 2, the bolded value expresses the maximum profit among all discussed methods and the related technique is indicated in the last row.

To more accurately evaluate the POP results, three days of the test period with different price behaviors have been considered. The actual clearing prices realized in the market are mostly higher than, mostly lower than, and close to the mean values of the expected ranges in these three test days, which are January 18, April 14, and July 7, respectively. It is worth to mention that these evaluations can be generalized as all days can be categorized into the above-mentioned three types of days.

Tables 3 and 4 show the impact of different DR values on the POP results of the RO approaches for the three days in 2014 in NYISO. By decreasing DR from \(\text{DR}^{\text{max}}\) to \(0.25 \times \text{DR}^{\text{max}}\) (in the steps of \(0.25 \times \text{DR}^{\text{max}}\)), the conservatism level of each RO approach reduces. The following points can be seen from Tables 3 and 4:

1. On the first test day (i.e. January 18), the actual clearing prices realized in the market are mostly higher than the mean values of the expected ranges. In this case, the less/more conservative methods lead to higher/lower profit. Therefore, the least conservative approach, i.e. BRO, obtains the highest profit on this test day.
2. On the second test day (i.e. April 14), the actual clearing prices are mostly lower than the mean values of the expected ranges. On this test day, the less/more conservative methods lead to lower/higher profit and the most conservative approach, i.e. PRO, obtains the highest profit.
3. On the third test day (i.e. July 7), the actual clearing prices are close to the mean values of the expected ranges. On this day, ERO, which is less conservative than PRO and more conservative than BRO, leads to the highest profit.
4. BERO/BPRO are less conservative than ERO/PRO and more conservative than BRO.

Tables 5 and 6 demonstrate the actual profit of BRO, ERO, PRO, CVaR-SP, and IGDT obtained from POP considering different price forecast errors. Note that according to Appendix C, BRO, BERO, and BPRO lead to the same profit for \(\text{DR} = \text{DR}^{\text{max}}\).

In each column of Tables 5 and 6, the bolded value shows the maximum profit among all non-deterministic approaches and the associated method is indicated in the last row. Based on Tables 5 and 6, it is concluded that:

1. The actual profit of the deterministic model is not always less/more than that of the non-deterministic models in all cases. The deterministic actual profit in several cases is higher/lower than the
Table 1
Actual profit of deterministic and robust approaches for four months using the IEEE 30-bus test system ($).  

<table>
<thead>
<tr>
<th>App.</th>
<th>January</th>
<th>April</th>
<th>July</th>
<th>October</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic</td>
<td>51,089,643</td>
<td>10,685,023</td>
<td>4,046,865</td>
<td>3,287,281</td>
</tr>
<tr>
<td>BRO</td>
<td>52,708,930</td>
<td>11,529,019</td>
<td>4,308,318</td>
<td>3,400,501</td>
</tr>
<tr>
<td>ERO</td>
<td>51,822,839</td>
<td>10,765,150</td>
<td>4,224,054</td>
<td>3,381,691</td>
</tr>
<tr>
<td>PRO</td>
<td>50,705,777</td>
<td>10,430,887</td>
<td>3,957,208</td>
<td>3,271,716</td>
</tr>
<tr>
<td>BRO</td>
<td>52,708,930</td>
<td>11,529,019</td>
<td>4,308,318</td>
<td>3,400,501</td>
</tr>
<tr>
<td>PRO</td>
<td>52,708,930</td>
<td>11,529,019</td>
<td>4,308,318</td>
<td>3,400,501</td>
</tr>
<tr>
<td>CVaR-SP</td>
<td>60,425,869</td>
<td>10,625,768</td>
<td>4,130,990</td>
<td>3,673,875</td>
</tr>
<tr>
<td>IGDT</td>
<td>44,342,860</td>
<td>9,743,124</td>
<td>3,187,930</td>
<td>2,783,550</td>
</tr>
</tbody>
</table>

Selected

| CVaR-SP | BRO/BRO | BRO/BRO | BRO/BRO | BRO/BRO |

Table 2
Actual profit of deterministic and robust approaches for four months using the IEEE 118-bus test system ($).  

<table>
<thead>
<tr>
<th>App.</th>
<th>January</th>
<th>April</th>
<th>July</th>
<th>October</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic</td>
<td>471,258,168</td>
<td>61,072,434</td>
<td>18,601,805</td>
<td>15,178,593</td>
</tr>
<tr>
<td>BRO</td>
<td>474,637,872</td>
<td>62,772,835</td>
<td>20,595,063</td>
<td>15,927,308</td>
</tr>
<tr>
<td>ERO</td>
<td>472,270,721</td>
<td>62,241,532</td>
<td>19,527,073</td>
<td>15,624,733</td>
</tr>
<tr>
<td>PRO</td>
<td>470,798,539</td>
<td>60,612,528</td>
<td>18,337,130</td>
<td>14,680,805</td>
</tr>
<tr>
<td>BRO</td>
<td>474,637,872</td>
<td>62,772,835</td>
<td>20,595,063</td>
<td>15,927,308</td>
</tr>
<tr>
<td>PRO</td>
<td>474,637,872</td>
<td>62,772,835</td>
<td>20,595,063</td>
<td>15,927,308</td>
</tr>
<tr>
<td>CVaR-SP</td>
<td>564,601,839</td>
<td>61,962,364</td>
<td>19,317,962</td>
<td>15,405,367</td>
</tr>
<tr>
<td>IGDT</td>
<td>352,420,273</td>
<td>54,543,388</td>
<td>17,133,489</td>
<td>14,027,273</td>
</tr>
</tbody>
</table>

Selected

| CVaR-SP | BRO/BRO | BRO/BRO | BRO/BRO | BRO/BRO |

Table 3
Actual profit of RO approaches vs. DR for the IEEE 30-bus test system ($).  

<table>
<thead>
<tr>
<th>Day</th>
<th>App.</th>
<th>$\Delta R_{Max}$</th>
<th>$0.75 \times \Delta R_{Max}$</th>
<th>$0.50 \times \Delta R_{Max}$</th>
<th>$0.25 \times \Delta R_{Max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 18th</td>
<td>BRO</td>
<td>329,503</td>
<td>337,932</td>
<td>345,425</td>
<td>351,324</td>
</tr>
<tr>
<td></td>
<td>ERO</td>
<td>317,545</td>
<td>323,951</td>
<td>332,168</td>
<td>341,873</td>
</tr>
<tr>
<td></td>
<td>PRO</td>
<td>283,910</td>
<td>305,980</td>
<td>314,775</td>
<td>323,031</td>
</tr>
<tr>
<td></td>
<td>BRO</td>
<td>329,503</td>
<td>335,962</td>
<td>339,571</td>
<td>343,320</td>
</tr>
<tr>
<td></td>
<td>PRO</td>
<td>329,503</td>
<td>334,194</td>
<td>336,042</td>
<td>342,882</td>
</tr>
<tr>
<td>April 14th</td>
<td>BRO</td>
<td>515,700</td>
<td>512,371</td>
<td>510,148</td>
<td>507,274</td>
</tr>
<tr>
<td></td>
<td>ERO</td>
<td>491,519</td>
<td>517,854</td>
<td>514,125</td>
<td>512,969</td>
</tr>
<tr>
<td></td>
<td>PRO</td>
<td>525,437</td>
<td>522,761</td>
<td>519,186</td>
<td>516,316</td>
</tr>
<tr>
<td></td>
<td>BRO</td>
<td>515,700</td>
<td>513,410</td>
<td>511,845</td>
<td>510,199</td>
</tr>
<tr>
<td></td>
<td>PRO</td>
<td>515,700</td>
<td>514,872</td>
<td>512,768</td>
<td>511,623</td>
</tr>
<tr>
<td>July 7th</td>
<td>BRO</td>
<td>430,610</td>
<td>431,485</td>
<td>432,811</td>
<td>433,328</td>
</tr>
<tr>
<td></td>
<td>ERO</td>
<td>437,693</td>
<td>438,729</td>
<td>440,004</td>
<td>441,983</td>
</tr>
<tr>
<td></td>
<td>PRO</td>
<td>424,853</td>
<td>425,982</td>
<td>428,952</td>
<td>429,250</td>
</tr>
<tr>
<td></td>
<td>BRO</td>
<td>430,610</td>
<td>432,942</td>
<td>434,710</td>
<td>435,532</td>
</tr>
<tr>
<td></td>
<td>PRO</td>
<td>430,610</td>
<td>432,254</td>
<td>433,558</td>
<td>434,390</td>
</tr>
</tbody>
</table>

Table 4
Actual profit of RO approaches vs. DR for the IEEE 118-bus test system ($).  

<table>
<thead>
<tr>
<th>Day</th>
<th>App.</th>
<th>$\Delta R_{Max}$</th>
<th>$0.75 \times \Delta R_{Max}$</th>
<th>$0.50 \times \Delta R_{Max}$</th>
<th>$0.25 \times \Delta R_{Max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 18th</td>
<td>BRO</td>
<td>1,698,159</td>
<td>1,731,237</td>
<td>1,781,518</td>
<td>1,797,305</td>
</tr>
<tr>
<td></td>
<td>ERO</td>
<td>1,619,552</td>
<td>1,662,324</td>
<td>1,699,042</td>
<td>1,746,495</td>
</tr>
<tr>
<td>April 14th</td>
<td>BRO</td>
<td>3,216,165</td>
<td>3,186,802</td>
<td>3,114,267</td>
<td>3,093,979</td>
</tr>
<tr>
<td></td>
<td>ERO</td>
<td>3,247,017</td>
<td>3,226,484</td>
<td>3,206,519</td>
<td>3,173,068</td>
</tr>
<tr>
<td>July 7th</td>
<td>BRO</td>
<td>3,474,114</td>
<td>3,309,194</td>
<td>3,246,771</td>
<td>3,220,232</td>
</tr>
</tbody>
</table>

Table 5
Actual profit of robust approaches vs. forecast error for the IEEE 30-bus test system ($).  

<table>
<thead>
<tr>
<th>Forecast error</th>
<th>App.</th>
<th>January 18th</th>
<th>April 14th</th>
<th>July 7th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic</td>
<td>BRO</td>
<td>304,440</td>
<td>516,092</td>
<td>432,120</td>
</tr>
<tr>
<td></td>
<td>ERO</td>
<td>343,585</td>
<td>515,700</td>
<td>430,610</td>
</tr>
<tr>
<td>Original error</td>
<td>BRO</td>
<td>296,910</td>
<td>525,437</td>
<td>424,853</td>
</tr>
<tr>
<td></td>
<td>CVaR-SP</td>
<td>318,724</td>
<td>519,186</td>
<td>435,934</td>
</tr>
<tr>
<td></td>
<td>IGDT</td>
<td>280,265</td>
<td>521,883</td>
<td>421,785</td>
</tr>
</tbody>
</table>

Selected

| CVaR-SP | CVaR-SP | CVaR-SP | CVaR-SP |

Selected

| CVaR-SP | CVaR-SP | CVaR-SP | CVaR-SP |

Selected

| CVaR-SP | CVaR-SP | CVaR-SP | CVaR-SP |

2x Original error

| CVaR-SP | CVaR-SP | CVaR-SP | CVaR-SP |

Selected

| IGDT | IGDT |

SP obtains the highest profit in the three test days. This can be explained as follows. The performance of CVaR-SP depends heavily on its scenarios. When the price forecast errors become half, much more accurate scenarios can be constructed and so the performance of CVaR-SP greatly improves. Indeed, with half forecast errors, the performance of all non-deterministic approaches of Tables 5 and 6 improve, but the amount of improvement of CVaR-SP is more than the other methods such that it becomes the most profitable method in this case.

4. If the price forecast errors doubles, i.e. if 200% less accurate price forecast method is used, the performance of all non-deterministic approaches of Tables 5 and 6 degrades. In this case, which is just the opposite of the previous case, the performance of CVaR-SP degrades more than the other approaches such that it obtains the lowest profits. On other hand, the IGDT method that gained the lowest improvement in the previous case, suffers from the lowest degradation in this case, since it is the most robust non-deterministic
approach in Tables 5 and 6. The IGDT method builds offer curves for the greatest level of uncertainty which guarantees the profit does not become worse than a definitive threshold.

The above points can help a GenCo select the most advantageous self-scheduling approach based on the price information and price forecast method that it has as well as the robustness level that it desires. The offer curves of unit 39 in hour 10 on January 18th, April 14th, and July 7th are illustrated in Fig. 2. These offer curves are associated with the results of the original forecast error of the IEEE 118-bus test system presented in Table 6. The offer curves in Fig. 2 demonstrate that BRO, PRO, and ERO compared to other approaches obtain highest profits for unit 39 in hour 10 on January 18th, April 14th, and July 7th, respectively.

| Table 6 | Actual profit of robust approaches vs. forecast error for the IEEE 118-bus test system ($). |
|-----------------|-----------------|-----------------|-----------------|
| Forecast error | App. | January 18th | April 14th | July 7th |
| Deterministic  | BRO  | 1,407,548     | 3,225,150 | 1,972,900 |
|                 | ERO  | 1,698,159     | 3,216,165 | 1,946,799 |
|                 | PRO  | 1,619,552     | 3,247,017 | 2,131,164 |
|                 | CVaR-SP | 1,269,494 | 3,347,114 | 1,881,772 |
|                 | IGDT | 1,475,327     | 3,238,478 | 2,057,170 |

<table>
<thead>
<tr>
<th>Selected</th>
<th>BRO</th>
<th>PRO</th>
<th>ERO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic</td>
<td>1,886,543</td>
<td>4,745,230</td>
<td>2,445,980</td>
</tr>
<tr>
<td>ERO</td>
<td>1,755,845</td>
<td>4,878,188</td>
<td>2,284,994</td>
</tr>
<tr>
<td>PRO</td>
<td>1,362,530</td>
<td>4,674,811</td>
<td>2,044,199</td>
</tr>
<tr>
<td>CVaR-SP</td>
<td>2,075,485</td>
<td>5,043,600</td>
<td>3,408,220</td>
</tr>
<tr>
<td>IGDT</td>
<td>1,240,546</td>
<td>4,664,170</td>
<td>1,991,751</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>0.5x Original error</th>
<th>Selected</th>
<th>BRO</th>
<th>PRO</th>
<th>ERO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic</td>
<td>987,860</td>
<td>2,847,950</td>
<td>1,114,752</td>
<td></td>
</tr>
<tr>
<td>ERO</td>
<td>1,008,476</td>
<td>2,908,185</td>
<td>1,136,588</td>
<td></td>
</tr>
<tr>
<td>PRO</td>
<td>1,067,628</td>
<td>2,971,476</td>
<td>1,164,949</td>
<td></td>
</tr>
<tr>
<td>CVaR-SP</td>
<td>1,015,623</td>
<td>2,938,120</td>
<td>1,150,422</td>
<td></td>
</tr>
<tr>
<td>IGDT</td>
<td>1,142,334</td>
<td>3,046,693</td>
<td>1,377,099</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2x Original error</th>
<th>Selected</th>
<th>BRO</th>
<th>PRO</th>
<th>ERO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic</td>
<td>1,407,548</td>
<td>3,225,150</td>
<td>1,972,900</td>
<td></td>
</tr>
<tr>
<td>ERO</td>
<td>1,698,159</td>
<td>3,216,165</td>
<td>1,946,799</td>
<td></td>
</tr>
<tr>
<td>PRO</td>
<td>1,619,552</td>
<td>3,247,017</td>
<td>2,131,164</td>
<td></td>
</tr>
<tr>
<td>CVaR-SP</td>
<td>1,269,494</td>
<td>3,347,114</td>
<td>1,881,772</td>
<td></td>
</tr>
<tr>
<td>IGDT</td>
<td>1,475,327</td>
<td>3,238,478</td>
<td>2,057,170</td>
<td></td>
</tr>
</tbody>
</table>

Table 7 compares the computation time of different robust methodologies for one scheduling day. Table 7 shows that (a) the MILP approaches of BRO, PRO, BPRO, and CVaR-SP have low computation times in the range of 160–334 s, (b) the MISOCP approaches of ERO and BERO have higher computation times of 3556 s and 7355 s, (c) the MINLP approach of IGDT has the highest computation time of 24,729 s based on the complexity of their models.

Table 8 presents a summary of the self-scheduling strategies for different real-market-environment cases. The self-scheduling strategies can help GenCos in making the most appropriate self-scheduling decision based on the price information and price forecast method that they have adopted as well as the robustness level that they desire in their solution. In Table 8, “Actual” term refers to the actual clearing prices realized in the market and “Mean Value” term refers to the mean values of the expected ranges. Note that since price forecast error typically occurs in all hours of a day, it is assumed in the RO models $\delta_{RD,R} = D_{RD,R}^{\text{max}}$, and according to Appendix C, BRO, BERO, and BPRO lead to the same profit for $D_{RD,R} = D_{RD,R}^{\text{max}}$. Fig. 2. Offer curves of unit 39 in hour 10 on January 18th, April 14th, and July 7th.
4. Conclusion

Various non-deterministic self-scheduling approaches have been presented in recent years. These approaches are mostly based on RO methodologies, SP frameworks and IGDT models. Which of the self-scheduling methods is appropriate for a specific GenCo depends on many factors including the company’s objectives, model complexity, available data, and computation time. The main motivation of this study was to evaluate and compare the performance of various robust self-scheduling methodologies. These robust methodologies have different uncertainty modeling approaches, as well as a range of tools for controlling the conservativeness of their solutions.

In addition to recasting the robust methodologies in more applicable forms, offer curve constructing strategy for each method has been presented. Moreover, to practically evaluate the performance of various methodologies, a POP has been proposed to determine the actual profit of each method in different real-market-environment cases. The conclusions drawn from the evaluations can help GenCos select and model the most appropriate non-deterministic self-scheduling approach based on the price information and price forecast method that they have adopted, as well as the robustness level that they desire in their solution.

Analyzing and comparing the performance of the non-deterministic approaches for multi-auctions self-scheduling model as well as considering different sources of uncertainty, such as electricity market price, renewable resources and so on, are set aside to be studied as future work.

Appendix A

The quadratic generation cost function $f_u(.)$ is as follows:

$$f_u(p_u) = a_u + b_u \cdot p_u + c_u \cdot p_u^2$$ \quad \forall u, \forall t \quad (A.1)

By applying the piecewise linear approximation \[20\] to (A.1), the linearized generation cost function is obtained in (A.2)-(A.6).

$$f_u(p_u) = A_u \cdot z_{ut} + \sum_{l=1}^{N_l} (S_{ul} \cdot p_{lul})$$ \quad \forall u, \forall t \quad (A.2)

$$p_{lut} = p_{ul}^\text{min} \cdot z_{ut} + \sum_{l=1}^{N_l} p_{lul} \quad \forall u, \forall t \quad (A.3)$$

$$p_{b(l-1)ul} \leq p_{ul}^{l-1} - p_{ul}^\text{min}$$ \quad \forall u, \forall t \quad (A.4)$$

$$p_{b(l)ul} \leq p_{ul}^l - p_{ul}^{l-1}$$ \quad \forall u, \forall t, \forall l = 1, \ldots, N_u^l \quad (A.5)$$

$$p_{b(N_l)ul} \leq p_{ul}^{\max} - p_{ul}^{N_u - 1}$$ \quad \forall u, \forall t \quad (A.6)$$

where $A_u = a_u + b_u \cdot p_{ul}^\text{min} + c_u \cdot (p_{ul}^{\min})^2$ \quad \forall u.

Appendix B

See Tables B.1 and B.2.

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>$p_{ul}^\text{min}$ (MW)</th>
<th>$p_{ul}^\text{max}$ (MW)</th>
<th>Cost coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$a_u$ ($/h$)</td>
</tr>
<tr>
<td>1</td>
<td>50</td>
<td>200</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>80</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>35</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
<td>30</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>12</td>
<td>40</td>
<td>0</td>
</tr>
</tbody>
</table>
Similarly, for (28), the following relation in the optimal solution is found:

\[
\sum_{i=1}^{T} (\nu + h_t) = \sum_{i=1}^{T} \sum_{u=1}^{U} E_{t,i} p_{u,t}
\]  

(C.2)
The above relations (C.1) and (C.2) can be written as:

\[
\sum_{t=1}^{T} h_t + \sum_{q=1}^{Q} q_t = \sum_{t=1}^{T} \sum_{u=1}^{U} E_t p_{ul}
\]

(C.3)

\[
T \cdot v + \sum_{t=1}^{T} h_t = \sum_{t=1}^{T} \sum_{u=1}^{U} E_t p_{ul}
\]

(C.4)

Based on DR = DR\text{max}, there are \(\Psi_b = 1\), \(\Psi_c = \sqrt{|J|} = \sqrt{T}\) and \(\Psi_p = |J| = T\) in BERO, and BPRO. Thus, the PF of (24) and (27) become:

\[
\left[\Psi_{\text{br}} \cdot \sum_{t=1}^{T} h_t \right] + \left[\Psi_{\text{cr}} \cdot \sqrt{\sum_{t=1}^{T} q_t^2} \right] = \sum_{t=1}^{T} h_t + \sqrt{T} \cdot \sqrt{\sum_{t=1}^{T} q_t^2}
\]

(C.5)

\[
[\Psi_{\text{pr}}] + \left[\Psi_{\text{pr}} \cdot \sum_{t=1}^{T} h_t \right] = T \cdot v + \sum_{t=1}^{T} h_t
\]

(C.6)

From Cauchy-Schwarz inequality, it is well-known that (38):

\[
(a, b) \leq (a, a)^{1/2} (b, b)^{1/2}
\]

(C.7)

where \((\ldots)\) is the inner product. Consider \(a = [1, 1]\), \(b = [q_1, q_2]\), and based on constraint (25),

\[
q_1 + q_2 \leq \sqrt{2} \cdot \sqrt{q_1^2 + q_2^2}
\]

(C.8)

This result can easily be extended to \(T\) terms as:

\[
\sqrt{T} \cdot \sqrt{\sum_{t=1}^{T} q_t^2} \geq \sum_{t=1}^{T} q_t
\]

(C.9)

Therefore, for DR = DR\text{max} to obtain the maximum value of (24), the ERO term \(\left(\sqrt{T} \cdot \sqrt{\sum_{t=1}^{T} q_t^2} \right)\) equals \(\sum_{t=1}^{T} q_t\). Using this result, combining (C.3) with (C.5) and (C.4) with (C.6) yield:

\[
\left[\Psi_{\text{br}} \cdot \sum_{t=1}^{T} h_t \right] + \left[\Psi_{\text{cr}} \cdot \sqrt{\sum_{t=1}^{T} q_t^2} \right] = \sum_{t=1}^{T} \sum_{u=1}^{U} E_t p_{ul}
\]

(C.10)

\[
[\Psi_{\text{pr}}] + \left[\Psi_{\text{pr}} \cdot \sum_{t=1}^{T} h_t \right] = \sum_{t=1}^{T} \sum_{u=1}^{U} E_t p_{ul}
\]

(C.11)

The right-hand-side of (C.10) and (C.11) is the last term of (13) for \(\Psi_b = 1\). Thus, for DR = DR\text{max}, the BRO, BERO, and BPRO models have the same PF, and thus lead to the same optimal result. Fig. C.1 illustrates the uncertainty set of BRO, BERO, and BPRO for DR = DR\text{max}. □
Appendix D

Theorem. In terms of conservativeness, BRO, ERO and PRO are sorted as: BRO \(\leq ERO \leq PRO\).

proof. For \(DR = DR^{max}\) the PF of the ERO in (16) becomes:

\[
\Psi_{\text{ERO}} = \sqrt{T} \sum_{t=1}^{T} \left[ E_t \sum_{u=1}^{U} P_{ud} \right] \leq \sqrt{T} \sum_{t=1}^{T} \left[ E_t \sum_{u=1}^{U} P_{ud} \right] = \Psi_{\text{ERO}}^{max}
\]

Using (C.8), for this PF it can be written that:

\[
\sqrt{T} \sum_{t=1}^{T} \left[ E_t \sum_{u=1}^{U} P_{ud} \right] \geq \sum_{t=1}^{T} \left[ E_t \sum_{u=1}^{U} P_{ud} \right] \geq \sqrt{T} \sum_{t=1}^{T} v^2 = T \cdot v
\]

The right-hand-side of (D.2) is the PF of BRO in (13) for \(DR = DR^{max}\), i.e. \(\Psi_{\text{Bro}} = 1\). Thus, ERO has a higher PF than BRO and so is more conservative than BRO for \(DR = DR^{max}\).

Considering the inequality \(v \geq \sum_{t=1}^{T} E_t \sum_{u=1}^{U} P_{ud}\) in (20), the PF of the ERO can be written as:

\[
\sqrt{T} \sum_{t=1}^{T} \left[ E_t \sum_{u=1}^{U} P_{ud} \right] \leq \sqrt{T} \sum_{t=1}^{T} v^2 = T \cdot v
\]

For \(DR = DR^{max}\), the PF of the PRO in (19) becomes:

\[
\Psi_{\text{PRO}} = T \cdot v
\]

(D.3) and (D.4) yield that PRO has a higher PF than ERO, and so is more conservative than ERO for \(DR = DR^{max}\). This result and the previous one proves the theorem for \(DR = DR^{max}\). Similarly, this theorem can be proved for the other values of \(DR\).

References