Approach for self-healing resilient operation of active distribution network with microgrid

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Abstract: Self-healing capability as one of the most important features of smart distribution networks (SDN) causes an increase in the resiliency and flexibility of networks by responding fast and restoring service during an outage. A two-layer algorithm based on metaheuristics is proposed for the optimal operation of smart distribution network in self-healing mode considering microgrid (MG) and direct load control (DLC) programme capability in the presence of micro turbine (MT) and energy storage (ES) in the distribution network. In the first layer, a new approach based on the graph theory is proposed to find the optimal sets of renewable energy resources \{WT, PV\} and FLV at the modified IEEE 33 bus distribution network.

Nomenclature

Sets

- $i$ set of nodes
- ES set of energy storage units
- $R$ set of renewable energy resources \{WT, PV\}
- FLV set of switches in $i$th fundamental loop

Parameters

- $S_{\text{max}}$ maximum transformer substation capacity
- $P_{d,i}^\text{ES}, Q_{d,i}^\text{ES}$ predicted active/reactive power output of renewable distributed generation (DG) at node $i$ in $R$ nodes at time $t$
- $P_{d,i}^\text{ES}, Q_{d,i}^\text{ES}$ active/reactive demand at node $i$ at time $t$
- $V_{\text{min}}$, $V_{\text{max}}$ minimum/maximum voltage magnitude constraint at node $i$
- $c_{\text{f}}, c_{\text{ij}}$ buying/selling price from/to upstream grid at time $t$
- $t_{\text{f}}, t_{\text{fmax}}$ minimum/maximum current magnitude constraint at brunch between nodes $i$ and $j$
- $t_{\text{f}}$ time of fault occurrence
- $t_{\text{c}}$ time of fault clearance
- $V_{i}, V_{ij}$ voltage magnitude at node $i$ at time $t$
- $P_{\text{ch}}, P_{\text{disch}}$ maximum charge/discharge rate ES at bus $i$
- $\eta^\text{ES}$ charging/discharging efficiency of ES at bus $i$
- $\text{SOC}^\text{ES}, \text{SOC}_{\text{max}}^\text{ES}$ minimum/maximum ES state of charge at node $i$
- $W_{b,i}$ capacity of ES at bus $i$
- $t_{\text{h}}$ optimisation horizon time
- $a^G_i, b^G_i, c^G_i$ MTs cost function coefficients
- $N_{\text{FL}}$ number of fundamental loops

Variables

- $SS_{i}$ selected switch from $i$th fundamental loop
- $x_{i,j,t}$ binary decision variable for feeder $ij$ at time $t$ (disconnected = 0, connected = 1)
- $P_{d,i}^\text{ES}, Q_{d,i}^\text{ES}$ active/reactive power buying/selling from/to upstream grid at time $t$
- $L_{c,i}$ load curtailment at node $i$ at time $t$
- $\mu_i, \lambda_i$ buying/selling state of DSO
- $\delta_{ij,t}$ phase difference between $V_{i}, V_{ij}$
- $w_{i}, \phi_{i}$ charging/discharging state of ES at bus $i$ at time $t$
- $c_{\text{f},i,t}$ connection state of load at node $i$ at time $t$ (shed = 0, connect = 1)
- $P_{d,i}^\text{ES}, Q_{d,i}^\text{ES}$ active/reactive power generation at node $i \in \text{MT}$ nodes at time $t$
- $P_{d,i}^\text{ES}$ active power output of ES at node $i \in \text{ES}$ nodes at time $t$
- $I_{ij,t}$ current magnitude at brunch between node $i$ and $j$ at time $t$

1 Introduction

Integrating high penetration of renewable energy sources (RES) in distribution network would bring many benefits to customers, utilities, and the nation such as power loss reduction, decrease in environmental pollution level, investments deferral, reliability indices improvement etc. [1, 2]. Microgrids (MGs) are introduced to address the emergence of high-penetration RES in distribution networks and are further identified as valuable alternatives to centralised generation and bulk transmission in power systems operation and planning [3].

On the other hand, high penetration of RES in distribution networks and significant impacts of events on electric power systems have resulted in the necessity of addressing the issue of distribution network resiliency [4]. Resiliency represents the ability of power systems to withstand low-probability high-impact events in an efficient manner while ensuring the least possible interruption in supply of electricity, and enabling a quick recovery and restoration to the normal operation state [5].

A novel and viable solution to address this issue is to use the MGs capability, especially in the event management [5]. Furthermore, communication development, automatic control infrastructures and installation of smart devices such as advanced metering and switches as well as increasing the penetration of DER play a significant role in the events management of distribution network, especially by adding self-healing capability [6]. Self-healing capability as one of the most important features of smart distribution network causes an increase in the resiliency and flexibility of networks.

MG forming is an important tool, which is capable of using the abovementioned resources for outage management in the...
emergency condition in order to increase network resiliency. As IEEE standard 1547.4 points out, operation and reliability of the distribution network can be improved by dividing the distribution system into multiple MGs [7]. After a fault occurrence, sectionalising switches will separate faulted zone and, consequently, form MGs. MGs can be operated in island mode or grid-connected mode during an outage occurrence. MGs which are located in the upstream of the network will stop taking service from the grid and act as a standalone source of power supply for their customers. In this mode, if the upper limit of MGs generation is smaller than momentary consumption, distribution system operator (DSO) will have the ability to perform direct load control (DLC) programmes by using communication infrastructures in order to maintain real-time balance between the generation and consumption [7]. This process is called distribution system restoration (DSR) [8]. As a result, forming of MGs is one the most important tools to optimise DSR, which can have a significant impact on improving the network resiliency.

Considering what was mentioned above, finding the optimal DSR strategy under the penetration of MGs and DERs is a fundamental task. DSR problem, as a non-linear issue, has numerous topological and operational constraints. For addressing the DSR problem, several approaches have been introduced such as expert systems [9], mathematical programming [11], multi-agent systems [12], fuzzy logic [13] and heuristic search [14, 15]. Bi-directional electrical power flow, meshed configurations, DERs operational constraints and protection logic are huge challenges in the way of determining the optimal DSR strategy [16]. Among these researches, there are few studies in which DSR strategy has been determined under consideration of MGs capability. In [17], a fully decentralised multi-agent system has been proposed to handle the complex DSR problem incorporating DERs. By dividing the distribution system into multiple MGs, Arefifar et al. [18] introduced an optimal model to increase the reliability. An agent-based paradigm for self-healing protection system with a graph theory-based expert system has been introduced by Sheng et al. [19]. Pham et al. [20] have developed a new DSR procedure based on graph models and a knapsack problem formulation using the dispersed generation availability. Liu et al. [21] have presented a framework for analysing the resilience of an electric power grid with integrated MGs in extreme conditions. The objective of this paper was to demonstrate that controllable MGs can help improve the resiliency of power grids in extreme conditions. A resilience-oriented DSR method using MGs to restore critical load has been developed in [22]. In this regard, a two-stage heuristic has been developed for the critical load restoration problem. First, a strategy table containing the information of all feasible restoration paths has been established. Then, the critical load restoration strategy has been obtained by solving a linear integer programme.

Also, through penetration of MG into the distribution network, many solutions have been proposed to meet the energy management challenges. A model for optimal scheduling of MGs considering multiple islanding constraints has been developed by Khodaei [23]. Wang et al. [24] introduced a decentralised coordinated energy management system of networked MGs in a distribution system considering the uncertainty of DG outputs and load consumption. Asimakopoulou et al. [25] have presented a leader–follower model and a bi-level programme for the energy management of networked MGs. The authors in [26] considered optimising the operation stage through some control variables, including load shedding at each bus, but without representing the objective function as a monetary value.

In [27, 28], multi-MG scheduling framework has been introduced by dressing up a self-healing mode. The main focus of these papers is on the MGs scheduling. DSR strategy and the way of dividing the distribution system into multiple MGs in the self-healing mode are pre-determined. In [16], a graph search-based algorithm has been proposed for smart distribution network (SDN) under the presence of MGs. In [29–31], a mathematical method is used to form MGs at the time of fault occurrence. Due to the considerable number of binary and continuous decision variables in the DSR problem, in [29, 30] a method has been proposed to reduce the problem space, thereby reducing the computation time of the problem. Yuan et al. [31] present a novel modified algorithm to identify the optimal distribution system restoration plan for improving the grid resiliency. An improved flexible switching pair operation is employed to maintain the radial nature of distribution system. Thus, the several studies which have been conducted in this area can be divided into two categories. The first group of papers have proposed methods for switching and forming of MGs at the time of fault occurrence with different objectives, including reducing outage duration and interrupted load as well as number of switching. The main focus of these papers is only on finding the optimal switching and forming of MGs at the time of fault occurrence. The other papers have assumed that the switching status and network configuration are already specified and have focused on the optimal operation of DERs. This is while, the pre-determined configuration is not necessarily the optimal one at the time of fault occurrence. The optimal configuration can be changed depending on the fault location, DERs generation, etc.

In this paper, considering the capability of MGs in improving the network resiliency, a novel approach for self-healing resilient operation of distribution network is represented in which the optimal DSR strategy including the optimal formation of MGs and DER generation is determined at the time of fault occurrence by using a two-layer metaheuristic-based algorithm. Considering a specific fault, the optimal formation of MGs is determined in the first layer and the optimal DER generation in the second layer. To do so, the first layer randomly generates the binary decision variables (the switching device status). As an initial population is randomly generated in the metaheuristic algorithm, many solutions are non-feasible due to operational constraint violations such as radial structure etc. In this situation, runtime will be greatly increased; however, fast decision making is a critical feature in SDN's self-healing scheme. So, the graph-based algorithm has been proposed to generate random binary decision variables to reduce the runtime of the method. Knowing the optimal configuration of the network, the optimal energy management programme is implemented in the second layer to determine the optimal operation of DERs including renewable energy resources, energy storages (ESs) etc. during the fault time. However, the optimal energy management schedule for 24 h is beyond the scope of this study, discharging of storage in the self-healing mode can affect the energy management after fault removal. Thus, operation cost after fault removal has been taken into account in DSR strategy. In summary, the proposed algorithm includes the following features:

i. Determining the optimal switching and DER optimal energy management in the self-healing intervals.
ii. Using graph theory to reduce the duration time of finding optimal DSR by creating a feasible configuration.
iii. Maximum network load restoration considering customers’ interruption cost.
iv. Taking into account the operation and load direct control programmes cost based on the customer damage function (CDF) in the self-healing mode and operation cost after fault removal in the DSR procedure.
v. Taking into account the network constraint by performing AC power flow.

This paper is organised as follows: Section 2 describes the proposed DSR strategy. In Section 3, problem formulation and constraints have been presented in two different operation modes including self-healing and normal operation. Numerical results are provided in Section 4. Section 5 has been assigned to conclusion.
substation which can be employed for data integration from the medium voltage feeder, DER and intelligent electronic devices (IEDs) and perform the control operation.

The proposed control structure is shown in Fig. 1. Data measured by IED are sent to CAMC in a carrier substrate. CAMC processes the received data, detects the operation mode of the network and updates the production levels of DER, the power purchased from the upstream grid, IED protection settings, network structure and controllable switches situation. Accordingly, CAMC plays a vital role in system monitoring, control and management. Definitely, the operator policies and objectives in a smart distribution network have a key role in CAMC processing and decision making [32].

In this paper, using the CAMC capability, a framework for monitoring and controlling the network in the normal and self-healing modes is proposed as shown in Fig. 2. In the normal situation, the decision variables are determined with different objectives such as economic benefit maximisation, reducing operation risk etc. [32]. With the fault occurrence, CAMC is able to propose the optimal manoeuvre point, network configuration changes, and forming of MGs with the aim of reducing the network outage and increasing the economic benefit. By sending IED’s data, the time, situation and type of the fault will be detected. By considering fault location and controllable switches, the smallest possible section of the network will be isolated. Then, the distribution network will be divided into several MGs according to the cost of operation in self-healing mode, customer damage cost and DLC programme.

The level of available production of dispatchable resources, SOC and the estimated production of non-dispatchable resources are taken into account in optimal MGs formation. Furthermore, loads, lines thermal limit, voltage and current limits as well as the presence of the DER with frequency control capability in each MG (because each MG must have at least one resource with frequency control capability) are considered, too.

After the formation of MGs, the optimal operation of DER is determined. Maintaining the generation and load balance, DLC programme is implemented considering customer damage cost. CAMC determines the DLC implementation process and DERs operation in formed MGs in an optimisation process. In the next step, CAMC updates the IEDs protection setting by taking into account the new network configuration and commitment generation production. The new configuration will be maintained until total elimination of fault. After fault removal, the network configuration returns to the previous structure in normal mode and the IEDs protection settings are updated according to the new operation mode.

This paper focuses mainly on providing the optimal solution for optimal forming of MGs and DER resource management in the self-healing mode at the time of fault occurrence. In this regard, it is assumed that with the availability of IED information and being processed by CAMC, the fault location is identified. According to the online process of operation at the real time, renewable and load uncertainties are ignored and optimal self-healing strategy is calculated based on the predicted values. In addition, in the restoration and MG formation processes, the transient changes are ignored. It should be noted that it is assumed that the network operator is the owner of the network and all of DERs.

3 Proposed approach

In this paper, a two-layer algorithm is proposed for optimal operation of distribution network in self-healing mode at the time of fault occurrence. The aim of the first layer is to determine the optimal forming of MG, and the aim of the second layer is to determine the optimal operation of MG, which is created in the first layer. Network optimal configuration, which is determined in the first layer, and the customer outage cost of the second layer, have a mutual effect on each other’s. The proposed method is shown in Fig. 3 and described in the following.

3.1 Optimal forming of MG

The optimal forming of MG is determined by the following steps.

3.1.1 Create possible network structure using graph theory. In fault occurrence, fast performance in SDN restoration is a characteristic of self-healing. Therefore, calculation execution time has special importance in these studies. In metaheuristic algorithms, initial population generation of possible network structure is done randomly. In these circumstances, a large number of structures are not feasible due to operational constraints violation such as radial constraint and the necessity of the presence of micro turbine (MT) with the capability of frequency control in
each MG. Also, the presence of MG in self-healing mode and distributed generation in SDN leads to an increase in the complexity of the restoration problem. In these circumstances, execution time will be increased, while in the smart grids, recovery process should take place as soon as possible.

Considering these issues, in this paper, graph theory is used to create the possible forming of MGs in the fault occurrence condition. According to this theory, each network is modelled with an equivalent graph as shown in Fig. 4. Each edge of this graph includes network controllable switches and the source node, consists of feeders which are connected to the upstream network, or the MT unit with the capability of frequency control. As an example, in Fig. 4a, MT units located at buses 12 and 15 have frequency control capability. Then the fundamental loops of the graph are determined. Fundamental loop consists of a set of basic simple (the smallest possible loop) and independent loops [33]. Each of the fundamental loops is described with a vector whose members are edges of the constituent loop, which are named fundamental loop vectors (FLVs).

As an example the equivalent graph shown in Fig. 4a has four fundamental loops (C1–C4). Each of these loops consists of a set of controllable switches as follows:

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**Fig. 3** Proposed framework

**Fig. 4** Network graph model

(a) Normal mode,
(b) Self-healing mode
Fundamental loops can be divided into real and virtual fundamental loops which are resulted from network structure and MT unit with consideration of network radial constraint and necessity of all load supply. A spanning tree is a sub-graph which includes all of the nodes of the graph, with minimum possible number of edges. This means that by removing an edge form a spanning tree, at least a couple of nodes that were originally connected get disconnected. The definition of spanning tree does not merely apply to fully connected graph. When applied to disconnected graph, it often takes the name of spanning forest [33]. In this paper, spanning forest is equivalent to an MG creation in the network. As an example, by selecting any common edge, all of the edges, which are located on the path of selected edge and loop subscription, should be removed from edge selection candidates in the next common loops. As an example, by selecting switch 10 from FLV, its intersection with the other FLV sets is investigated. Considering switch 10 intersection with FLV, and FLV, FLV is updated and SS will be randomly selected from the following set:

\[
\text{FLV}_1 = \{10, 11, 12\}
\]

\[
\text{FLV}_2 = \{1, 2, 3, 4, 5, 6, 8, 9, 10, 11\}
\]

\[
\text{FLV}_3 = \{1, 2, 3, 13, 14, 16, 17, 18\}
\]

\[
\text{FLV}_4 = \{15, 16, 17, 18\}
\]

Random spanning forest created

**Fig. 5 Graph theory algorithm**

\[
\text{FLV} = \{10, 11, 12\}
\]

\[
\text{FLV} = \{1, 2, 3, 4, 5, 6, 8, 9\}
\]

\[
\text{FLV} = \{1, 2, 3, 13, 14, 16, 17, 18\}
\]

\[
\text{FLV} = \{15, 16, 17, 18\}
\]

This rule ensures that the created structures do not have any loops and all loads are connected to the generation point. In addition, one of the remarkable features of the proposed method is the capability of creation of desired number of islands.

In this situation, if only one switch is selected from each fundamental loop, the network will be integrated in the form of spanning tree. Selecting each switch from each virtual loop is equivalent to an MG creation in the network. As an example, by selecting switches 17 and 10 from the fundamental loops and 4 and 1 leads to creation of MG1 and MG2, respectively. While by closing switch 17, MG1 will connect to feeder F2 and only MG2 will remain in the network. In contrast, by closing switch 10, MG2 will be connected to feeder F3 and MG3 will just be created in self-healing mode.

In addition to the above two cases, neglecting both virtual fundamental loops and closing switches 10 and 17 lead to spanning tree and integrated structure. Therefore, by considering or neglecting virtual loop in the process of random selection of opened switches, the operator can control the number of created MGs. The notable point is that by considering fault section isolation of network, one of the opened switches has always been considered as a certain value and hence, the number of variables will be decreased.

This proposed method for random generation of possible MG structure in self-healing mode is represented in Fig. 5. These configurations include a certain number of desirable islands in accordance with the radial constraints and considering the presence of at least one MT unit with the capability of frequency control.

### 3.1.2 Find MGs optimal formation using particle swarm optimisation (PSO)

Knowing the possible configuration of network, a PSO algorithm is used to determine the optimal one. Furthermore, the optimal implementation of DLC programme is implemented. Decision variables in the self-healing mode include controllable switches situation for optimal MG formation, controllable resources power and storage resources generation, storage charge and discharge mode and controllable load mode. Cost function includes customer damage cost, DLC implementation cost and operation cost. On the other hand, storage's charge/discharge programme in self-healing mode affects the network operation after fault removal. Therefore, the operation cost in the hours after fault removal has been considered in the objective function. This causes the ESs in MGs, which have extra generation, not to go to the discharge mode so that their energy is maintained for the hours after fault removal. Without considering operation cost in the hours after fault removal, the result of optimal operation will always be ESs discharge in each MG. After fault removal, network structure returns to the optimal structure before fault occurrence and the optimal energy management programme is implemented. Due to the impact of storage discharge in restoration time on the next hour energy management, the operation cost after fault removal is added to the cost function. The objective function is expressed as follows: (see (1)) such that

\[
\mu_i + \lambda_i \leq 1 \quad \forall i
\]

\[
(P_{\text{DSO}}^{\text{PSO}}) + (Q_{\text{DSO}}^{\text{PSO}}) \leq (S_{\text{MAX}})
\]
The first term of (1) includes costs of MT resources operation, electricity purchase from upstream network and DLC programme implementation. The DLC programme implementation cost is calculated based on CDF in the self-healing mode in the time period of fault occurrence until its removal. The second term includes the above mentioned cost in the time period in the normal operation after fault removal. In (2), \( \mu \) and \( \lambda \) are binary variables which determine the electricity trade with the upstream network. Constraint (3) bounds maximum capacity of sub transmission system shown in Fig. 6.

\[
P_{i,t} = V_{i,t} \sum_{j} V_{i,j}x_{i,j}(G_{i,j}\cos \delta_{i,j} + B_{i,j}\sin \delta_{i,j}) \quad \forall \, i,t \tag{4}
\]

\[
Q_{i,t} = V_{i,t} \sum_{j} V_{i,j}x_{i,j}(G_{i,j}\sin \delta_{i,j} - B_{i,j}\cos \delta_{i,j}) \quad \forall \, i,t \tag{5}
\]

\[
P_{i,t} = p_{i,t}^{ES} + p_{i,t}^{PS} + p_{i,t}^{DLC} - cs_{i,t}p_{DLC}^{PS} \quad \forall \, i,t \tag{6}
\]

\[
Q_{i,t} = q_{i,t}^{ES} + q_{i,t}^{PS} - cs_{i,t}q_{DLC}^{PS} \quad \forall \, i,t \tag{7}
\]

\[
I_{i}^{max} \leq I_{i,t} \leq I_{i}^{min} \quad \forall \, i,t \tag{8}
\]

\[
-P_{i,t}^{ch,\max} \psi_{i,t} \leq P_{i,t}^{ES} \leq P_{i,t}^{ch,\max} \phi_{i,t} \quad \forall \, i \in ES \, nodes, \, t \tag{9}
\]

\[
SOC_{i,t} = SOC_{i,t-1} - \left( \frac{\psi_{i,t}P_{i,t}^{ES}}{W_{i,t}} \right) \quad \forall \, i \in ES \, nodes, \, t \tag{10}
\]

\[
SOC_{i,t}^{\max} \leq SOC_{i,t} \leq SOC_{i,t}^{\min} \quad \forall \, i \in ES \, nodes, \, t \tag{11}
\]

\[
(P_{i,t}^{ES})^2 + (Q_{i,t}^{ES})^2 \leq S_{i,t}^{max} \quad \forall \, i \in WTs \, nodes, \, t \tag{12}
\]

The optimisation problem was solved by using DICOPT solver [34].

4 Numerical results

The proposed method is assessed using the 33-bus distribution system shown in Fig. 6a. All of the network data is considered as [35].

Time variation of PV and wind generation are considered as multiples of their capacity as shown in Fig. 7a. Active and reactive profile of customer and energy price at the main bus are shown in Figs. 7b and c. The data of PV, wind MT and storage unit are provided in Tables 1–3, respectively. The maximum SOC and \( P_{i,t}^{DLC} \) are considered 0.9 and 0.95, respectively. Customer damage cost is presented in Table 4. Customers located at buses 5, 7, 8, 24, 25, 30 and 32 are considered non-residential.

In this paper, it is supposed that the two MT units which are located at 19 and 25 buses have the capability of frequency and voltage control. Thus, considering the proposed method, the graph of studied network is as shown in Fig. 6b. As can be seen in this figure, the considered graph includes five real fundamental loops (C1–C5) and two virtual fundamental loops (C6 and C7). In these circumstances, in accordance with Section 4.1, the integrated structure is connected to the upper network, the structure with one and two MGs can be created in optimisation process. Based on the above information, case studies are examined for different scenarios as follows. The proposed algorithm is implemented on a PC with 3.4 GHz CPU and 16 GB RAM.

4.1 Scenario 1

In this scenario, it is supposed that a permanent fault occurs on the main feeder between buses 4 and 5 at 19 h and continues for 2 h. In this situation, regarding the primary structure of the network, outage occurs in the significant part of the network. With the implementation of the proposed algorithm, the network structure is restored to the structure with two MGs and one part is connected to the upstream network as shown in Fig. 8a. In this scenario, the system is restored by closing S33, S34, S35, S36 and S35 switches and opening S4, S24, S26, S30, S9, S12 and S16 switches. In self-healing mode due to network structure changes and creation of MGs, the MT and ES units will be redispached. Table 5 shows the MT generation dispatch, ES dispatch and optimal load shedding in each MG in scenario 1.

Table 5 shows the optimal load shedding and optimal dispatch of ESs, MTs and MGs in scenario 1.

4.2 Scenario 2

In this scenario, it is supposed that a permanent fault occurs on the main feeder between buses 4 and 2 at 19 h and remains for 2 h. By isolating the faulted part, all network loads will be interrupted. With the implementation of the proposed algorithm, the network structure is restored to structure with two MGs as shown in Fig. 8b. In this scenario, the system is restored by closing S33, S34, S35, S36 and S35 switches and opening S1, S29, S17, S12, S21, S20 and S3 switches. Network operation program in self-healing mode is represented in Table 6. Operation cost, customer outage cost, loss and minimum voltage of each MG in self-healing mode are shown in Table 6. As can be seen, in optimal strategy voltage constraint is met between 0.95 and 1.05.

As shown in Fig. 7, optimal strategy leads to restoration of all interrupted load and reduces customer supply cost in self-healing mode. As can be seen, storage located at MG2 is discharged to reduce customer damage cost. The SOC of ES at bus 33 is reduced to 0.249 at 19:59 and to zero at 20:59. While the other ESs, which are located at the connected part of the network, are charged because the price at 19 and 21 is low compared with 22 through 24. The SOC of ES5, ES14 and ES20 are, respectively, 0.559, 0.718 and 0.809 at 20:59. Thus, the operation cost after fault removal is the reason why ESs which are located at the upstream part of the network are not charged at the fault time and part of their energy is maintained for the time after fault removal.

\[
\min \sum_{i \in WTs} \sum_{t=1}^{T} \left( a_i^{C} + b_i^{C}p_{i,t}^{ES} + c_i^{C}(p_{i,t}^{ES})^2 + (1 - cs_{i,t})CDF_{i,t}LC_{i,t} + \left( \mu_{i,t} - \lambda_{i,t} \right)^2 P_{i,t}^{DLC} \right)
\]

\[
+ \sum_{i \in MG} \sum_{t=1}^{T} \left( a_i^{C} + b_i^{C}p_{i,t}^{ES} + c_i^{C}(p_{i,t}^{ES})^2 + (\mu_{i,t} - \lambda_{i,t})^2 P_{i,t}^{DLC} \right)
\]
Results show that MT units are operated at the maximum capacity in fault period. In this scenario, with fault occurrence at time 19 and due to power shortage, the load at bus 29 is interrupted. Load increment in time interval 20 is the reason, in addition to bus 29, the loads located at buses 3 and 23 are interrupted too. Also, unlike the first scenario, all ESs have changed to the discharge mode to create balance between generation and consumption. So the SOC of ES5, ES14, ES20 and ES33 are reduced to 0.05, 0.451, 0.251, respectively, at time 19:59 and reach 0 at the time of fault removal.

4.3 Computational performance

In this section, the results of two scenarios are compared with and without using graph theory to evaluate the quality of the proposed methodology and computational performance. Without using graph, the binary variables, including the status of network switches, are generated randomly and with heuristic search. The results of the two methods are presented in Table 9.

As expected, in the case of using heuristic search, many of the proposed configurations are unfeasible due to violation of radial constraint. So, in order to achieve the optimal solution, we need to increase the number of particle and repetition of PSO in the proposed method. In this case, the execution time of the method significantly increases and the chance of finding the optimal response is reduced. As shown in the above table, without using...
graph algorithms, execution time of method is increased from 15.3 to 943.1 s in the first scenario and the total cost, including the cost of resource operation and outage cost, is increased from 552.08 $ to 571.6 $. Also in the second scenario, the execution time of the method is increased from 16.9 to 1286.5 s and the total cost is raised from 1671.24$ to 1992.70$. Therefore, the proposed method

Fig. 7 Input data
(a) Estimated power of wind and PV units,
(b) Hourly multiplier load profile,
(c) Energy price at the main bus

Table 1 Non-dispatchable unit's data

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<thead>
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<th>Type</th>
<th>Bus</th>
<th>Size, kW</th>
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<td>PV</td>
<td>3, 9, 17, 28</td>
<td>60, 60, 40, 50</td>
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<tr>
<td>WT</td>
<td>4, 16, 22, 24, 26, 30</td>
<td>30, 85, 60, 50, 50, 70</td>
</tr>
</tbody>
</table>

Table 2 MT unit data

<table>
<thead>
<tr>
<th>Bus</th>
<th>(a_G), $</th>
<th>(b_G), $/kW</th>
<th>(c_G), $/kW²</th>
<th>(p_G,\text{max})</th>
<th>(p_G,\text{mic})</th>
<th>Power factor</th>
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<tr>
<td>15</td>
<td>0.025</td>
<td>0.071</td>
<td>0.00008</td>
<td>800</td>
<td>0</td>
<td>0.95</td>
</tr>
<tr>
<td>18</td>
<td>0.039</td>
<td>0.055</td>
<td>0.00004</td>
<td>650</td>
<td>0</td>
<td>0.95</td>
</tr>
<tr>
<td>19</td>
<td>0.023</td>
<td>0.062</td>
<td>0.00003</td>
<td>100</td>
<td>0</td>
<td>0.95</td>
</tr>
<tr>
<td>25</td>
<td>0.010</td>
<td>0.075</td>
<td>0.00006</td>
<td>750</td>
<td>0</td>
<td>0.95</td>
</tr>
<tr>
<td>29</td>
<td>0.010</td>
<td>0.061</td>
<td>0.00003</td>
<td>750</td>
<td>0</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Table 3 Energy storage data

<table>
<thead>
<tr>
<th>Bus</th>
<th>(W_{E,\text{in}}), kWh</th>
<th>(p_{\text{ch,max}}), kW</th>
<th>(p_{\text{dch,max}}), kW</th>
<th>Initial SOC</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>150</td>
<td>70</td>
<td>70</td>
<td>0.1</td>
</tr>
<tr>
<td>14</td>
<td>150</td>
<td>70</td>
<td>70</td>
<td>0.9</td>
</tr>
<tr>
<td>20</td>
<td>200</td>
<td>80</td>
<td>80</td>
<td>0.8</td>
</tr>
<tr>
<td>33</td>
<td>200</td>
<td>80</td>
<td>80</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 4 Customer damage cost ($/kWh)

<table>
<thead>
<tr>
<th>Duration</th>
<th>momentary</th>
<th>30 min</th>
<th>1 h</th>
<th>4 h</th>
<th>8 h</th>
</tr>
</thead>
<tbody>
<tr>
<td>residential, $</td>
<td>0.56</td>
<td>1.75</td>
<td>2.20</td>
<td>4.80</td>
<td>7.20</td>
</tr>
<tr>
<td>non-residential, $</td>
<td>53</td>
<td>198</td>
<td>282</td>
<td>1196</td>
<td>2369</td>
</tr>
</tbody>
</table>
leads to improvement of the accuracy and execution time of calculation by limiting the search space of PSO algorithm. This improvement is very important in outage management of electric distribution network which is obtained by using graph theory.

*Fig. 8 Optimal structure in self-healing mode*

(a) Scenario 1
(b) Scenario 2
In this paper, a two-layer algorithm based on the metaheuristics is proposed for the optimal operation of smart distribution network in self-healing mode. In the first layer, the optimal forming of MGs is
determined based on the graph theory. According to this method, the search domain for the optimal structure is reduced; resulting in significantly reducing the runtime calculations. In the second layer, the utilisation of RES and storage is taken into account as the other decision variables. The results have shown that storages have a significant role in reducing costs in self-healing mode. Meanwhile, the dependency of the operation on the costs in the hours after fault removal has shown that these costs should be considered in the optimal self-healing strategies. The proposed algorithm can be used for central control system with the high level of penetration of RES, widespread using of smart equipment and advanced control as well as adaptive control and protection scheme in smart distribution networks and also for reliability improvement.

6 References


Table 8 Operation cost, interruption cost, loss and minimum voltage in self-healing mode-Scenario 2

<table>
<thead>
<tr>
<th>Time</th>
<th>Network parts</th>
<th>Operation cost, $</th>
<th>Interruption cost, $</th>
<th>Loss, kW</th>
<th>Min voltage magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>19:00</td>
<td>upstream connected part</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>MG1</td>
<td></td>
<td>237.78</td>
<td>372.20</td>
<td>9.09</td>
<td>22</td>
</tr>
<tr>
<td>MG2</td>
<td></td>
<td>44.25</td>
<td>6.22</td>
<td>30</td>
<td>0.993</td>
</tr>
<tr>
<td>20:00</td>
<td>upstream connected part</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>MG1</td>
<td></td>
<td>237.21</td>
<td>733.01</td>
<td>8.17</td>
<td>22</td>
</tr>
<tr>
<td>MG2</td>
<td></td>
<td>46.79</td>
<td>6.34</td>
<td>30</td>
<td>0.993</td>
</tr>
<tr>
<td>total</td>
<td></td>
<td>566.03</td>
<td>1105.21</td>
<td>29.82</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 9 Comparison of operation solution

<table>
<thead>
<tr>
<th>Method</th>
<th>Scenario 1 (fault location: S4)</th>
<th>Scenario 2 (fault location: S1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>proposed approach</td>
<td></td>
<td></td>
</tr>
<tr>
<td>open switch</td>
<td>S4, S24, S26, S30, S9, S12, S16</td>
<td>S1, S29, S17, S12, S21, S20, S3</td>
</tr>
<tr>
<td>running time, s</td>
<td>15.3</td>
<td>16.9</td>
</tr>
<tr>
<td>total cost, $</td>
<td>552.08</td>
<td>1671.24</td>
</tr>
<tr>
<td>heuristic search</td>
<td></td>
<td></td>
</tr>
<tr>
<td>open switch</td>
<td>S4, S22, S26, S30, S10, S14, S16</td>
<td>S1, S32, S15, S12, S35, S33, S3</td>
</tr>
<tr>
<td>running time, s</td>
<td>943.1</td>
<td>1286.5</td>
</tr>
<tr>
<td>total cost, $</td>
<td>571.65</td>
<td>1992.70</td>
</tr>
</tbody>
</table>